

Research article

# DEVELOPMENT OF MATHEMATICAL MODEL TO MONITOR THE INFLUENCES OF PERMEABILITY AND POROSITY IN MASS CONCRETE FORMATION

Eluozo, S. N.

Subaka Nigeria Limited, Port Harcourt, Rivers State of Nigeria  
Director and Principal Consultant, Civil and Environmental Engineering, Research and Development  
E-mail: [Soloeluzo2013@hotmail.com](mailto:Soloeluzo2013@hotmail.com)  
E-mail: [solomoneluzo2000@yahoo.com](mailto:solomoneluzo2000@yahoo.com)

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## Abstract

Permeability and porosity were found to deposit a serious influence in microstructural properties of concrete formation, the micropores of concrete are determined by the structural characteristics known as fine and coarse aggregate, fine sand and water including different types of reinforcement components are determined depending on the type of impose load, compressive strength of concrete are base on mix design under the influences of water cement ratio, the concrete characteristics determined the percentage or degree of porosity and permeability, the transport and place of concrete determine the percentage of this deposition in concrete formation, several experimental methods has been applied to determine the rate of influence in concrete structure, but it is only little conceptualization in modeling of this established parameters in the system that has been done, base on these conditions, mathematical model to monitor the rate of both parameters were developed, this is to ensure the influences of these parameters in concrete structures are determined, the developed governing equation were modified to ensure that the rate of permeability and porosity are express mathematically, so that the rate of decrease and increase of compressive strengths can be monitored from this dimensions. **Copyright © IJMMT, all rights reserved.**

Keywords: mathematical model, permeability and porosity and mass concrete formation

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## 1. Introduction

In concrete, various flow mechanisms have different rates of transmission in a singular pore structure, and concrete pore structures undergo dynamic changes. As no two concrete samples are exactly alike, a broad classification was developed to categorize concrete into three groups. Table 5 classifies the porosity-permeation system, from low to

high porosity and the corresponding transfer mechanisms. The three systems in Table 5 represent increasing cementitious content and/or levels of hydration. The most porous system corresponds to high w/c ratios (above  $w/c = 0.7$ ) and/or low levels of hydration, where high porosity is compounded by high connectivity. In this system, most of the water is free and unaffected by the surface forces of the hydration products. As the internal flow paths are unobstructed, the pressure-induced flow would provide the highest transmission rates, through the pore structure. Most structural concretes are cast at u/c ratios of 0.45 to 0.65. The existing porosity (as determined by the initial w/c) is divided into a multitude of finer pores and, under a proper curing regime; complete discontinuity of the pore system is possible.

The materials used for permeable pavement are classified into nine categories (Ferguson 2005): porous aggregate, porous turf, plastic geocells, open-jointed paving blocks, open-celled paving grids, porous concrete, porous asphalt, soft paving materials, and decks. Concrete has been used in pavement surfaces since 1865, when dense concrete street pavements were first experimentally installed in Scotland (Croney 1997). Porous concrete was first used in pavements during World War II. As a subset of the broader family of permeable pavements, porous concrete is also referred to as permeable concrete, enhanced porosity concrete, or Portland cement pervious pavement. It is normally made of single-sized aggregate bound together by Portland cement, physically and chemically identical to dense concrete (Ferguson 2005). Permeable concrete is relatively porous, providing by the omission of fine aggregates (Scholz & Grabowiecki 2007) and filled most of volume with coarse aggregate, thus, porous concrete obtains more voids in the structure leading to Goodwater infiltration and air exchange rates. Different types of aggregates exhibit different strength, permeability and geometry stability due to different mineral composition, grain sizes, types of formation, texture and location of the aggregates source. Coarse aggregate is mainly used as a primary ingredient in making the pervious concrete. Fine aggregates were not added to the mixture in this research. According to Krezel (2006), crushed igneous rocks are more preferable as coarse aggregate for concrete due to their higher strength. However, since the availability of igneous rock in Australia is becoming scarce Krezel (2006), this research diverted to the crushed sedimentary and metamorphic rocks.

Research of pervious concrete has ever been conducted at Tennessee Technological University (Crouch et al. 2007). It is indicated that the compressive strength, effective void content and permeability are largely dependent upon the aggregate. Crouch et al. (2007) stated that not only the size of aggregate, but also the gradation and amount of aggregate could affect the compressive strength and static modulus of elasticity on pervious Portland cement concrete. Meininger (1988) used different aggregate sizes (10 mm and 19 mm) in nonfine concrete study and the results showed that larger aggregate sizes would result in lower compressive strength, which corresponded with the results found from Yang & Jing (2003). It claimed the decrease of aggregate size led to higher pervious concrete strength, resulting from the increase of the interface strength between the aggregate and cement paste (Yang & Jing 2003). Ghafoori and Dutta (1995) also set up the relationship between gravimetric air content and permeability and porosity in no-fines concrete. However, in Australia there has been no published research that reveals the effect of aggregates on the structural performance of pervious concrete.

## Theoretical background

Permeability and porosity in concrete formation has been observed to have a common relation in terms of structural formation, concrete characteristics of coarse, fine aggregate cement and water mix ratio in concrete mix design for any construction are determine through the rate of permeability and porosity in concrete structure, most case, reinforced material such as mild steel are found to play major role in terms of permeability and porosity percentage in concrete formation. The percentage of porosity is base on the micropores between concrete mixtures including the ease of water passing through the concrete influenced by the rate of microstructural deposition in concrete formation. Base on the dimension it is clear that both parameters have a common relationship. The concept of determining there influenced in compressive strength of concrete may have been expressed mathematically in this direction such condition is imperative to establish a mathematical model to express these influenced of porosity and permeability in concrete structure, this expression will monitor the variation under the influences of mix design, through the establishments of water cement ratio. Establish mathematical governing equation will express every condition both parameters play there various roles due to increase or decrease of compressive strength of concrete structures. The relationship between porosity and transport is of particular interest. Many tests that directly measure transport properties require specialized equipment and long periods of time to Complete (e.g., saturated water permeability). Powers and colleagues “observed that the degree of permeability was controlled mainly the capillary porosity” [power, 1959]. In the course of further hydration, the capillary pores become disconnected and the

permeability is controlled by the “gel pores.” Given that the capillary’ pore system presents the pathway for the ingress of deleterious substances, it follows that the formation of a discontinuous capillary pore system is highly desirable. Powers [1947] also suggests that moist curing of field concrete past the point of achieving discontinuity’ is of little value. The theories of permeability’ in porous media arise from two schools of thought. One is the application of the Poiseuille-Hagen law (used by Hughes [Hughes, 1985]; the other considers viscous drag of moving fluid on a particle. Powers and his coworkers [power, 1959] took the second approach, which uses Stoke’s law as a basis. (Stoke’s law is also used when determining particle size distribution of small particles using sedimentation, such as silt and clay.) As the concentration of particles increases, the Stoke’s velocity decreases. [Steiniou, 1944] determined a function for the variation in particle concentration using data derived from tapioca suspended in oil. The actual function used is proportional to the inverse of the hydraulic radius, dependent on particle concentration and temperature. The theory was extended to hardened cement paste since it is a porous solid with particle connections involving a small fraction of the surface, and can be thought as a collection of particles.

### 3. Governing Equation

$$\phi \frac{\partial c}{\partial t} = K \frac{\partial^2 c}{\partial x^2} \dots\dots\dots (1)$$

The governing equations express the rate of porosity and permeability relationship in concrete formation, \concrete characteristics are found to determine the influences of porosity and permeability deposit in concrete structures. Moreso it confirming the solid surface area calculated by the permeability relationship that express the rate of measured water by absorption. The expressed governing equations are derived to monitor the rate of porosity and permeability influence in concrete structural formation.

Substituting solution C = XT into (1), we have

$$XT^1 = KX^{11}T \dots\dots\dots (2)$$

$$\phi \frac{T^1}{T} = -K \frac{X^{11}}{X} T \dots\dots\dots (3)$$

$$\phi \frac{T^1}{T} - K \left( \frac{X^{11}}{X} \right) \dots\dots\dots (4)$$

$$\phi \frac{T^1}{T} - \frac{X^1}{X} \dots\dots\dots (5)$$

Considering when Ln x 0 →

$$\phi T^1 = K \frac{X^1}{X} - T = \lambda^2 \dots\dots\dots (6)$$

$$\phi \frac{T^1}{T} = \lambda^2 \dots\dots\dots (7)$$

$$\frac{X^{11}}{X} = \lambda^2 \dots\dots\dots (8)$$

$$K = \lambda^2 \dots\dots\dots (9)$$

This implies that equation (10) can be expressed as:

$$K \frac{X^{11}}{X} = \lambda^2 \dots\dots\dots (10)$$

$$K \frac{X^2}{X} = \lambda^2 \dots\dots\dots (11)$$

$$\phi \frac{dy^2}{dx^2} = \lambda^2 \dots\dots\dots (12)$$

$$\frac{Kdy^2}{dx^2} = \lambda^2 \dots\dots\dots (13)$$

$$\phi \frac{d^2y}{dx^2} = \lambda^2 \dots\dots\dots (14)$$

$$\frac{d^2y}{dx^2} = \frac{\lambda^2}{\phi} \dots\dots\dots (15)$$

$$d^2y = \left( \frac{\lambda^2}{\phi} \right) dx^2 \dots\dots\dots (16)$$

$$\int d^2y = \int \frac{\lambda^2}{\phi} dx^2 \dots\dots\dots (17)$$

$$dy = \frac{\lambda^2}{\phi} x dx \dots\dots\dots (18)$$

$$\int dy = \int \frac{\lambda^2}{\phi} X dx + C_1 \dots\dots\dots (19)$$

$$y = \frac{\lambda^2}{\phi} X^2 + C_1x + C_2 \dots\dots\dots (20)$$

$$y = \frac{\lambda^2}{\phi} + C_1x + C_2 \dots\dots\dots (21)$$

$$y = 0$$

$$\Rightarrow \frac{\lambda^2}{\phi} X^2 + C_1x + C_2 = 0 \dots\dots\dots (22)$$

The expression from equation [2-22] were able monitored the rate of porosity at several condition separating these parameter under the application of variables separation, this is were mathematical methods are generated, it showcase the reality under the influences of porosity and determined the rate of fluid flow in the concrete. Various flow mechanisms have different rates of transmission in a singular pore structure, and concrete pore structures undergo dynamic changes. Experimentally, no two concrete samples are exactly alike, a broad classification was developed to categorize concrete into three groups. As the internal flow paths are unobstructed, the pressure-induced flow would provide the highest transmission rates, through the pore structure. Most structural concretes are cast at u/c ratios of 0.45 to 0.65. The existing porosity (as determined by the initial w/c) that is divided into a multitude of finer pores and, under a proper curing regime; complete discontinuity of the pore system is possible, this concept are determined to monitor various rate of influences in concrete structures\, such condition determine the rate of compressive strength under the influences of water cement ratio through mix designed

Applying quadratic expression, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \dots\dots\dots (23)$$

Where a =  $\frac{\lambda^2}{\phi}$ , b = C<sub>1</sub> and c = C<sub>2</sub>

$$X = \frac{-(C_1) \pm \sqrt{(C_1)^2 - 4\left(\frac{\lambda^2}{\phi}\right)C_2}}{2\frac{\lambda^2}{\phi}} \dots\dots\dots (24)$$

$$= \frac{-C_1 \pm \sqrt{C_1^2 - 4C_2\frac{\lambda^2}{\phi}}}{2\frac{\lambda^2}{\phi}} \dots\dots\dots (25)$$

$$X = \frac{-C_1 + \sqrt{C_1^2 - 4C_2\frac{\lambda^2}{\phi}}}{2\frac{\lambda^2}{\phi}} \dots\dots\dots (26)$$

$$X = \frac{-C_1 + \sqrt{C_1^2 - 4C_2\frac{\lambda^2}{\phi}}}{2C_1} \dots\dots\dots (27)$$

$$X = \frac{-C - \sqrt{C_1^2 - 4C_2 \frac{\lambda^2}{\phi}}}{2 \frac{\lambda^2}{\phi}} \dots\dots\dots (28)$$

Substituting equation (20) to the following boundary conditions and initial values condition

$$t = 0 \quad C = 0 \dots\dots\dots (29)$$

Therefore,  $X_{(x)} = C_1 e^{-m_1 x} + C_2 e^{m_2 x} \dots\dots\dots (30)$

$$C_1 \cos M_1 x + C_2 \sin M_2 x \dots\dots\dots (31)$$

$$y = \frac{\lambda^2}{\phi} + C_1 + C_2 \dots\dots\dots (32)$$

$$C(x,t) = \left( C_1 \cos M_1 \frac{\lambda^2}{\phi} x + C_2 \sin M_2 \frac{\lambda^2}{\phi} x \right) \dots\dots\dots (33)$$

But if  $x = \frac{v}{t}$

Therefore, equation (33) can be expressed as:

$$C(x,t) = \left( C_1 \cos M_1 \frac{\lambda^2 v}{\phi t} + C_2 \sin M_2 \frac{\lambda^2 v}{\phi t} \right) \dots\dots\dots (34)$$

The final expressed derived solution is the developed mathematical model that will monitor the rate of permeability and porosity in concrete formation. Several experiments application has been in done in this area, but there has not been any thorough sequential mathematical model to showcase there established relationship in this direction, base on this development mathematical equation were modified to monitor the rate of both parameter in concrete formation, this expressed model considered the major influential parameters and boundary conditions, which has produces the expressed condition mathematically in the derived solution. Moreso Currently, porosity and absorption measurements, such as C 642 is commonly used. Rate of absorption measurements, such as C 1585, are useful to determine the rate of uptake of liquids into unsaturated concretes, and since they measure uptake from one face, they are useful for evaluation-of the combined effects of compaction, composition. this curing history on an exposed surface. For vapor transmission through concrete exposed to liquid water on one side, E96 can be used. To measure surface moisture emissions from floor slabs, F 1869 has been used for many years and F 2170 has been more recently developed. These tests were developed to determine when it was suitable to apply floor finishes, but have been also used to estimate vapor transmission rates through concrete. Chloride ingress by diffusion (C 1556) and combined transport mechanisms (C 1543) exist. The C 1556 test, when run for a range of chloride exposure periods, can provide a bulk chloride diffusion value and time-dependent change for input into some service-life prediction models. As rapid indices of penetration resistance, the C 1202 test continues to be useful for quality control and quality assurance purposes, although other tests, such as AASHTO TP 64 and resist's its/conductivity tests may prove to be more useful in the future. Progress will continue to be made on test methods, due to the impetus to produce highly durable concrete structures, and to more accurately predict their service lives. This concept from exponential level has be done, but this study has provide mathematical representations considering condition that are integrated considering condition that has been expressed in other previous results

#### 4. Conclusion

As stated in the preface, the notion of stability encompasses both porosity and permeability characteristics of concrete. Porosity in concrete formation is complex and covers pores ranging from nanometer to centimeter sizes. Dimension of porosity is considerably influenced by sample history and conditioning. Also, the rate of concrete porosity is not a static value and can concurrently undergo development during the hydration process and decline in extreme exposures. Dimensions of mass transfer through this continually changing pore structure become a function of testing parameters and the type of transport that is being tested. There have been significant advances in the theories and test methods involving the measurement of transport mechanisms and porosity in the intervening years, but mathematically there is no sequential conceptualization of porosity and permeability in concrete formation influence in compressive strength variation, this condition called for the establishment of these relationships through the development of mathematical model. The developed model considered all the influential parameters in the system in modifying the derived solution that established the model that will monitor the rate of influences in porosity and permeability in concrete formation.

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